Pulling the Goalie: Hockey and Investment Implications

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In 2016, *Sports Illustrated* named the 1980 "Miracle on Ice" as the greatest moment in sports history.[4] A collection of US college hockey players from the University of Minnesota and Boston University, average age 21, beat the Soviet Union team on the way to winning an Olympic gold medal. The Soviets won every other Olympic gold hockey medal from 1964 to 1992 (outlasting the Soviet Union itself), and most international matches it played over 30 years.

Team USA went up 4-3 with ten minutes to play in the third period. The Russian team's furious attacks were held off, and the Americans even got some solid shots on goal. But as the minutes ticked down, legendary Russian coach Victor Tikhonov did not pull his goalie to get a sixth attacker. Russian defenseman Sergei Starikov later explained that, "We never did six-on-five," not even in practice, because, "Tikhonov just didn't believe in it."[5,6]

Pulling the goalie is one of the more dramatic moves in hockey, traditionally credited to coach Art Ross[7] in a 1931 playoff game between his Boston Bruins and the Montreal Canadiens. Down 0-1 with a minute to play, he sent goaltender Tiny Thompson to the bench and inserted a sixth attacker. It didn't work, there was no more scoring and Boston lost. But a tradition was born.[8]

Actually, two traditions were born, one good, one not so much. Pulling the goalie is a sound strategic move, but waiting until there is one minute left to play is not. Only recently have hockey coaches begun to pull goalies earlier, and it's still nowhere near early enough.[9]

There have been a number of papers published on the subject using different models and data, but all agree that goalies should be pulled earlier than is the usual practice.[10] Some, like ours, say it should be much earlier. Some of these models consider factors such as which team is better, who has the puck, what zone a faceoff is in, penalty situation, home or road game, among others. However, the focus of these papers is on hockey game advice. We do that too but that's not our main goal. We consider the problem of optimal decision making in a broader strategic context, not to inform hockey coaches (though we'd echo the unanimous advice of this set of papers that they're not pulling their goalies nearly enough, our model suggests pulling on the early side of all this research that recommends earlier

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[4] *Sports Illustrated, https://www.si.com/specials/100-greatest/?q=1-miracle-on-ice.*

[5] See Coffey (2005).

[6] An alternative explanation is they just had very little experience with trailing in games!

[7] The same Art Ross who donated the Art Ross trophy given each year to the NHL player who scores the most points.

[8] See Zweig (2015).

[9] We discuss some reasons for this later. It is not that NHL coaches don't understand the game.

[10] In an Appendix, we discuss in more depth some of these papers and models, and how they relate to ours.

action), but to illuminate how decisions are made and to recommend how to improve them. We apply these recommendations to our chosen fields of investing, business, and risk management.

We build a model that uses five inputs: the probability of scoring goals with a goalie in place, with the goalie pulled for an extra attacker, with a goalie in place but the other team has pulled their goalie for an extra attacker, the goal differential, and the time remaining in the game. This has fewer game-level parameters (3) and game situation parameters (2) than other models.[11] It's not meant for precise calculation of real situations in individual games, but for assessing long-term average decision-making.[12] Proceeding in this way creates a simple model with attractive intuition, which also yields many interesting comparative statics.[13]

For the probability of scoring goals while at full strength for both teams, we took the number of such goals scored in the 2015-16 NHL season (4,947) and divided by the total number of minutes played at that strength (126,425) to get an average rate of 0.0391 per team per minute (4.69 total even strength goals per game). We treated that as a constant for our analysis and broke it down into a 0.65% chance (.0391 divided by six) of scoring at even strength in each 10 second interval, ignoring the negligible possibility of multiple scores within the same 10 second interval. Using the same methodology for the 2,206 minutes played during the season with both teams at full strength when one has pulled the goalie, we get a rate of 1.18% per 10 second interval for the team with the sixth attacker[14] and 2.58% for the team that retains its goalie.

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[11] It might seem as if we need to include the relative values of wins, ties and losses. We don't. In the current NHL regular season, a team gets 2 points for a win, 1 for a loss in overtime and 0 for a loss. That makes a tie at the end of regulation worth about 1.5 points, or 75% of a win (assuming equal chances for both teams in overtime). In the playoffs however, and in other leagues, a tie at the end of regulation might be worth only half as much as a win. But we consider only situations in which a team is behind, in which case either they will lose the game, or get to a tie at some point. Since the decision to pull the goalie can be reversed if the game becomes tied, all that we need to know is that a tie score is better than being behind. How much better doesn't matter. Well, actually, it matters to how much this all matters, but not to optimal decision making.

[12] Tactical game situations lend more richness to the decision making process. For instance, Zaman (2001) finds that you'd pull earlier when the puck is in the offensive zone (a faceoff being the clearest example) and later for defensive zone draws (when goalies are typically put back in, though we'd guess too automatically especially as the clock really ticks down), and, less obvious, how big is the score differential. This seems quite intuitive as the probabilities of scoring for each team, that we use as constant inputs, would conditionally be different. So, previewing our final result, we find that the goalie should be pulled much earlier than current convention, but as we consider average situations, not specific ones, it's likely even earlier for offensive zone draws.

[13] Among other things we assume two average teams on scoring and defense, both sides playing at full strength, the puck in neither offensive zone and that there is no momentum in goal scoring. These assumptions greatly simplify the analysis without changing the major factors and, most importantly, do not affect the decision making takeaways we apply to other areas beyond hockey.

[14] 1.18% per 10 seconds corresponds to a 13.3% success rate on a full uninterrupted two-minute power play. This is slightly less than you normally see for power play effectiveness, likely resulting from 6 on 5 being a smaller advantage than 5 on 4. Another possible explanation is that the short-handed team in the NHL scores on only about 2% of power plays, while in our model there is a 27% chance the team with a goalie in place will score within two minutes. In both NHL power plays, and in our model, the team with an extra attacker for two minutes scores about 25% as often as when nobody scores.

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That immediately shows that pulling the goalie is a negative expected value move in terms of goals (as 1.18% is a lot lower than 2.58%). The team that pulls its goalie nearly quadruples the probability of its opponent scoring, while not even doubling its own chance to score. But expected value of goals is not the appropriate criterion. What matters is expected number of standings points. A team down a goal with short time remaining gains a lot by scoring, and loses little if the other team scores as losing by two goals is no worse than losing by one (admittedly our model doesn't consider pride!).[15] Perhaps the first lesson that we can apply outside of hockey is that sometimes what seems like the right criterion is, in fact, not, and selecting the right one can make an important difference. We discuss this idea more later on.

To solve for the optimal strategy, we begin by defining the expected point function of never pulling your goalie in any situation $EP_{NP}(score\ differential, time\ remaining)$. Call this the "Tikhonov" option.[16] It has two parameters, score differential, S, and time remaining in the game, t. We assume a loss is worth zero standing points, a win is worth two, and a tie at the end of regulation 1.5 (only the 1.5 is an assumption, the others are simply known). We know that:

$EP_{NP}(S, 0)$ is zero for $S < 0$, 1.5 for $S = 0$ and 2 for $S > 0$ (the values only affect the nominal calculations, not the decision on when to pull the goalie when behind) and that

$$EP_{NP}(S, t) = 0.9870 EP_{NP}(S, t - 10) + 0.0065 EP_{NP}(S + 1, t - 10) + 0.0065 EP_{NP}(S - 1, t - 10)$$

This works because if we never pull the goalie, we assume each team has a 0.65% chance of scoring in each ten-second period, hence the two 0.0065s and 1.0000 - 0.0065 - 0.0065 = 0.9870, and the chances are exclusive. Working backwards from 10 seconds left (our equation is undefined at T=0 and is only meaningful at 10 second increments) these equations allow us to compute $EP_{NP}(S, t)$ for all S and t.

Next, we solve for the expected point functions if you pull now and act optimally thereafter, $EP_{PO}(S, t)$, and if you don't pull now and act optimally thereafter, $EP_{NO}(S, t)$ (EP_{NO} differs from EP_{NP} as while both don't pull the goalie now, EP_{NP} commits to never doing it while EP_{NO} doesn't pull now but will pull optimally going forward). The initial conditions are the same as for $EP_{NP}(S, 0)$: $EP_{PO}(S, 0) = EP_{NO}(S, 0)$ equals zero for $S < 0$, 1.5 for $S = 0$ and 2 for $S > 0$. We also know that:

$$EP_{PO}(S, t) = 0.9624 Max[EP_{PO}(S, t - 10), EP_{NO}(S, t - 10)]$$
$$+ 0.0118 Max[EP_{PO}(S + 1, t - 10), EP_{NO}(S + 1, t - 10)]$$
$$+ 0.0258 Max[EP_{PO}(S - 1, t - 10), EP_{NO}(S - 1, t - 10)]$$

The 0.9624 figure comes from the probability that with the goalie pulled nobody scores, which is one minus the sum of 1.18% plus 2.58% (recall we're assuming both things can't happen in the same ten

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[15] With the minor exception that the final tiebreaker for determining NHL playoff standings when two or more clubs are tied in points is the greatest differential between total goals for minus total goals against for the entire regular season. We don't incorporate this into our model.

[16] We've also built an additional mathematical model to help Coach Tikhonov: $EP_{Tikhonov}$(score differential, time remaining). It translates roughly as: NEVER PULL VLADISLAV TRETIAK FOR A DIFFERENT GOALIE AFTER JUST ONE PERIOD!!!!

second period). The total (EP$_{PO}$(S,t)) is the probability that nobody scores times keeping the goalie pulled going forward plus the probability you score and then you act optimally (the max of either pulling or not pulling going forward after scoring so it's S+1 not S); and the same idea for the bad event where the other team scores.[17]

This allows us to compute $EP_{PO}(S, t)$ and $EP_{NO}(S, t)$ for all S and t; and the maximum of those two for any S and t is the expected point value from following optimal pull strategy from that point forward. The optimal strategy is to pull whenever $EP_{PO}(S, t) > EP_{NO}(S, t)$. Again, we find all EP$_{PO}$ and EP$_{NO}$ values by working back in ten second instruments from the end of the game.[18]

Figure 1 shows the expected point advantage over never pulling from either pulling now, or not pulling now, and acting optimally thereafter when down one goal with the indicated time remaining in the game. The maximum advantage from optimal pulling comes at 4:20 and is just over 0.08 points (note, the time of maximum advantage over never pulling is not the optimal time to pull, which actually comes even earlier). While that may seem small it can be a significant difference over an entire season.

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[17] We write both the second and third terms as "max" functions of pulling or not pulling going forward, which is technically correct, but not really necessary if only down one goal as we know if we score (S+1) we won't pull (the goalie will return), and if we don't score the goalie will remain pulled. It's necessary for situations where the losing team is trailing by more than a goal.

[18] We believe this dynamic programming approach is the natural one for a problem where you are really certain what to do with very little time left (10 seconds in our model) and so can naturally work backwards using three well-defined natural states in each increment (you score, they score, or nobody scores). The most similar approach to ours is probably that of Erkut (1987). He works in continuous time, and thus gets a Poisson distribution for number of goals scored in a given interval, and derives an approximate closed-form expression for the optimal pull time as opposed to our iterative calculation. This should give quantitatively similar results to ours. The main difference is that he assumes a higher rate of goal scoring (his data came from an era when teams averaged nearly four goals a game, compared to about three today), and a particularly high rate of empty net scoring, based on a small sample of games. If we put his scoring averages into our model we get an optimal pull time down one goal of 3:10 versus his 2:50 for the lowest scoring rate he considers, and 1:40 matching his 1:40 for the highest scoring rate.

Figure 1: Expected point value advantage over never pulling
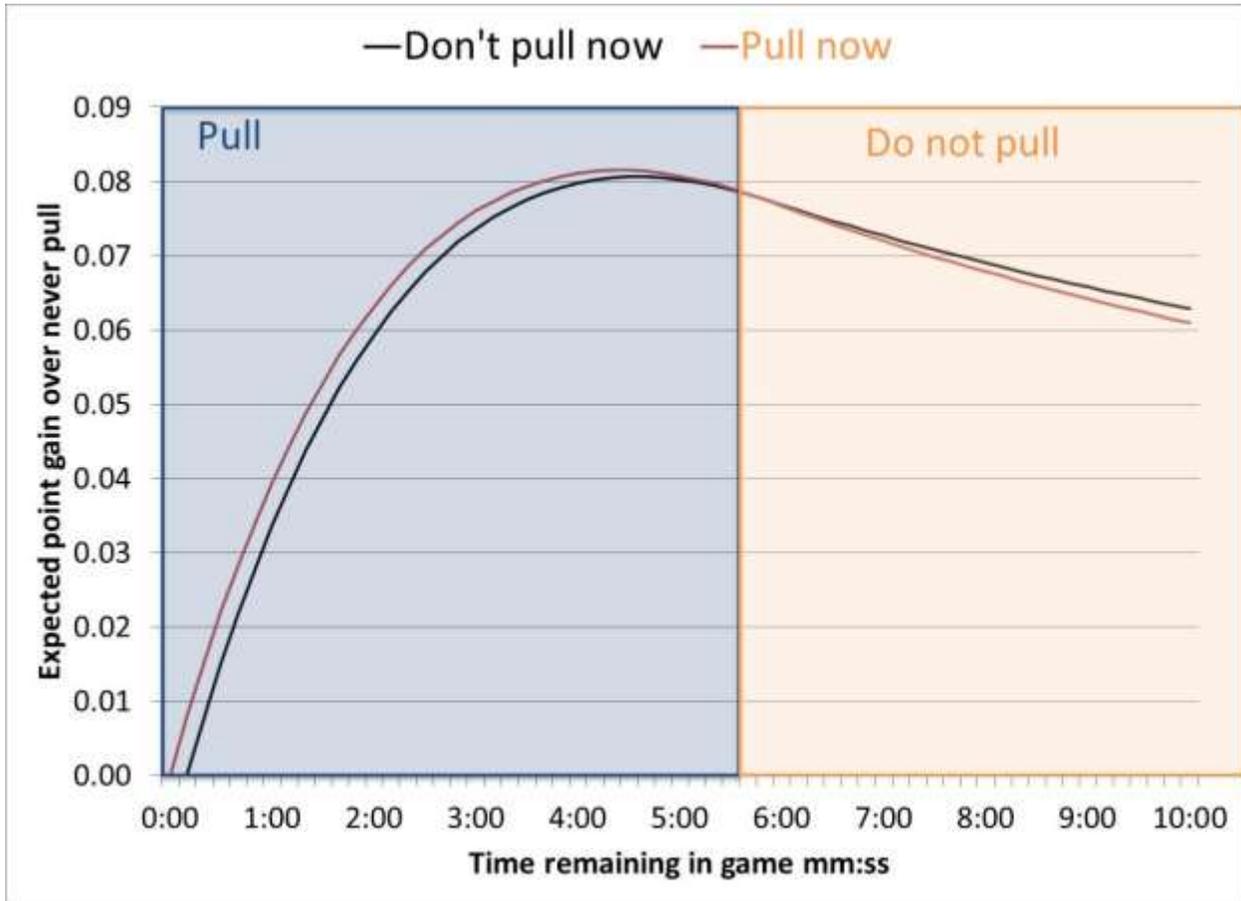
The crossover point comes at 5:40 remaining. So, at 5:50 you should not pull the goalie, but at 5:40 you should.

A team that practices optimal goalie pulling gains an average of 0.02 more points per game.[19,20] That is worth 1.76 points in an 82-game season, over a team that never pulls the goalie. In the 2015-16 season,

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[19] This figure comes from running our goal scoring model for the entire game. That assumes the teams are evenly matched and it ignores important factors like power plays and less-frequent scoring in the first period versus the second and third. While our model was not designed to be accurate for full-game purposes, it does give reasonable values for the frequency of games tied in regulation, decided by one goal and decided by more than one goal; these are the important parameters for determining the value of optimal pulling over a season. Since the best teams in hockey win about twice as often at the worst teams, they are likely behind near the end of the game roughly half as often, and therefore would get only about half the benefit from optimal pulling. If the average benefit is 0.02, we might guess that the benefit is 0.0133 for the best teams and 0.0266 for the worst teams.

[20] Another way to get to the 0.02 extra points per game figure is not to run our model over the whole game but to use our model just for the benefit of optimal pulling while trailing late (about 0.08 in standing points) and some additional rough estimates. This has the benefit of leaning far less on our model and using it when it's likely most accurate – in close games near the end. An average team loses by one about 12.5% of the time. That gets you (12.5% x 0.08) about 0.01 extra points over the season from pulling optimally in this situation. Another 25% of the time they lose by more than one goal. In this case they still get some (albeit much smaller) benefit from optimally pulling, and in some of those instances they were trailing by one and got an empty net goal against (adding

24 of the 30 NHL teams were closer than 1.76 points to either the team ahead of them in the standings or the team behind them, so this is a material difference in expected performance that comes without extra cost or work.

When down two goals, it pays to pull the goalie with 11:40 to go, less than halfway through the third period. If you score to make it one goal down, you replace your goalie until 5:40, as any earlier is too aggressive when only down by one. So, if you're still down by one goal at 5:40, you pull again.

When down three goals, you should pull at 17:50, just over two minutes into the third period; down four goals you should pull with 35:50 remaining; and down five goals or more, you can pull at any time. These numbers may sound silly, but they make sense and matter. A team that is down five goals early in the first period (we might argue in this case the goalie isn't doing any good anyway, but ignore that) still expects to collect 0.04 points on average, and can increase that 75% to 0.07 points by pulling the goalie immediately and keeping him out until the team pulls within four goals, and using the optimal rules thereafter. The intuition is the same as pulling near the end of the game. If you do nothing down five goals early your chance of winning is tiny and you have little to lose trying to climb back into the game (losing by six is not that much worse than losing by five). Though we'd admit the assumptions and simplifications of our model are probably being pushed much harder for this "losing by a ton early" analysis.

Let's make sure to stress the obvious (to hockey fans). When losing by one, we find that coaches should pull the goalie at the 5+ minute mark, not the 1+ minute mark common in the NHL, and the difference matters. Trailing by two, we find that the goalie should be pulled with more than 10 minutes to go, something you never see.

We can also use our model for some interesting comparative statics. If the league suddenly became much higher scoring, with all probabilities doubling,[21] then the optimal time to pull shortens to about two and a half minutes remaining (this would also apply to a particular game in the current league if the particular teams were both much more high scoring than average). If we enter a dark age for goal scorers, let's call this scenario "soccer," by say halving all scoring probabilities (down to only about 2+ even strength total goals a game), then a team trailing by one would pull the goalie with more than ten minutes left. The intuition is clear. If the chance of all scoring is much lower, then you have to pull earlier as the "safe is death" logic of pulling the goalie is even stronger because the chance that something happens, and you desperately need <u>something</u> to happen when trailing, is now much smaller if you do nothing. Interestingly, the total value of acting optimally vs. never pulling is reasonably stable to doubling or halving scoring (though in the high scoring scenario the cost of the NHL standard pulling at 1 to 1.5 minutes left is less suboptimal).

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additional one goal games beyond what we observe in the final score when optimal pulling policy would matter a lot). Finally, some fraction of overtime games were the result of a team trailing by one and scoring with their goalie pulled, and this is an additional situation when optimal pulling policy mattered. All considered, this rougher estimate makes us comfortable with our model's result of 0.02 extra points per game from optimal pulling policy.

[21] This equates to near ten even strength goals a game on average – think the 1980s Oilers for the whole league, or alternatively, the alternate reality where Jacques Lemaire followed his true calling and became a poet instead of a hockey coach.

Now, if we drop only the chance of scoring with the goalie pulled by 25% (from the assumed above 1.18% to 0.88%), and all else remains the same, you'd still optimally pull at about 2.5 minutes remaining. This means that NHL coaches are still pulling too late even if we significantly reduce the chance of it working. In fact, you have to reduce this chance by nearly 40% to make pulling at the one minute mark approximately the optimal strategy.

Now, let's get to our real purpose (besides just loving both hockey and math). There are some important risk management and investing lessons to be learned by considering this optimal hockey strategy problem. The most basic lesson is to make sure you are thinking about the right risk. Pulling the goalie always increases the volatility of numbers of goals scored, and is a negative expectation in terms of the score. For those reasons it is often used as a metaphor for a high-risk, desperation move. However, the point of hockey is not to maximize the differential between the goals your team scores during the season and the goals it gives up (if it were, no one should ever pull a goalie). The point of hockey is to maximize the number of standing points — a team down by a goal with short time remaining gains a lot by scoring, and loses little if the other team scores — which argues for a different measure of risk and return. As we have shown, pulling the goalie actually reduces the risk of losing the game — it's an insurance move — and this is the proper risk measure.

Investors sometimes make similar mistakes when they focus on the risk of an investment, rather than the risk an investment adds to their overall portfolio. They make this same mistake when they focus on the volatility of their portfolio rather than the probability of an unacceptable level of return over their risk horizon. If increased portfolio volatility comes with sufficient extra excess expected return, over long periods of time a portfolio can have a better chance of an acceptable return by taking more short-term volatility.

This issue of selecting the proper risk measure also relates to valuing a stock option. If a call option is well out-of-the-money with little time left to expiry, an investor would prefer that the volatility of the underlying asset increase because the now higher volatility increases the value of the option and thus reduces the risk of losing (as it reduces the likelihood of the option expiring worthless), even if it comes at the cost of a lower or negative expected return on the underlying asset. Pulling the goalie is effectively like an option holder being able to increase volatility on their own (at the cost of lowering the expected return if measured in terms of goals but not in terms of the truly relevant measure of points). There's no management lesson in this one, just a neat analogy.

Another lesson comes from asking why NHL coaches don't pull goalies earlier. This question relates to a [well-documented](#) phenomenon in sports.[22] Examples include, basketball coaches were very slow to have players shoot enough three point shots; football coaches don't go for it on fourth down nearly enough, nor attempt enough two point conversions; and baseball managers were very slow to appreciate the value of walks, the cost of outs, and the utility of the excruciatingly annoying radical infield shift. All these coaches face tremendous pressure to win, and they have the benefit of many repetitions of game

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[22] See Moskowitz (2011).

situations to get precise statistics to guide the right choice. Yet they consistently fail, often for decades, even after every numerate fan has figured things out.

Two reasons have been advanced for this failure to act. First, coaches are not actually rewarded for winning. They are rewarded for being perceived as good coaches. Obviously, the two are closely related but not exactly the same thing. If a basketball coach gets his team to execute crisp offensive plays with few turnovers that lead to two point baskets on 50% of possessions, he's deemed an excellent coach. If his team still loses 100 – 102, well, his players just weren't quite good enough. If the same coach encourages his team to "run and gun" threes, with lots of turnovers and misses, but scoring on 35% of possessions, he's clearly lost control of his team. If they win 105 – 102 it's perceived as just luck as everyone knows three point shots are risky. Essentially winning ugly is undervalued versus losing elegantly; and losing ugly can be career suicide. Once again, the way you measure risk matters in making the optimal decision.

This is a problem in portfolio management as well. A chief-investment officer (CIO) who runs a tight ship, putting his money with low-fee index funds and moderate fee active managers who beat their benchmarks, is perceived as an excellent manager (more so these days than in earlier times when perhaps the opposite, a CIO investing only in high fee ex post successful stock pickers, was conventional wisdom). Although he is indeed likely to be excellent as those are good things, they are nonetheless sometimes not good enough, in which case a CIO should look into alternative choices and new types of risk. For example, accepting some leverage risk for the benefit of additional portfolio diversification[23], or taking some liquidity risk in exchange for higher expected return. But if these things don't work out, the CIO can lose his reputation for competence; and if they do work out, well, everyone knows it was luck because all those things were risky. As John Maynard Keynes pointed out, "Worldly wisdom teaches that it is better for reputation to fail conventionally than to succeed unconventionally." This applies to both coaching sports and portfolio management.

We can also apply this lesson to security selection. Cheap stocks (using simple ratios like price-to-book or price-to-sales) tend to outperform expensive stocks. But they also tend to be "worse" companies — companies with less exciting prospects and more problems. Portfolio managers who own the expensive subset of stocks can be perceived as prudent while those who own the cheap ones seem rash. Nope, the data say otherwise.

The second reason coaches shy away from actions with short-term risk is that sins of commission are far more obvious than sins of omission.[24] The hockey coach who pulls his goalie down 0 – 2 with ten minutes to go and loses 0 – 5 will face harsh criticism from every quarter. A coach who quietly loses 1 – 2, pulling his goalie only in the final minute, can hold his head up and say his guys played hard but the puck didn't roll their way tonight; it was a close game, and they'll work even harder to get the breaks tomorrow (giving 110% of course).

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[23] Full disclosure, we're talking up one of the author's day jobs now!

[24] Omission vs. commission must be defined as versus the norm. Thus we're also talking about what's often called "maverick risk" in investing.

Unfortunately, CIOs face this issue as well. If a portfolio is hammered by overexposure to equity risk in a crash, well, the market went down, what can you do? His investment team worked hard, but the markets didn't roll their way this quarter (they failed conventionally), they'll work even harder to get the breaks next quarter. But a CIO who chooses an alternative investment that disappoints will face sharp criticism, even if the portfolio now has a better expected long-term ratio of risk to return (and succeeds unconventionally).

We can view the question of why coaches don't pull early enough in yet another way that has an investing analogy. Investors have been shown to be reluctant to sell their losers (part of the so-called "disposition effect") presumably as selling is psychologically "locking in" a loss. Might the extreme reluctance towards pulling the goalie, when say down two with more than ten minutes left, be the result of a similar cause? Pulling the goalie earlier may be the best action in terms of expected points but runs a very high probability of going down three goals, and thereby almost "locking in" the loss. Also, our model absolutely ignores that hockey is entertainment. Perhaps, more speculatively, fans enjoy the last 1-2 minute drama very much, and even will stay tuned to a game with a late two goal differential. But, pulling the goalie very early when down two goals leads to a very large chance of being down three. That would kill the entertainment. Might coaches feel a desire, or even tacit pressure, not to ruin the fun? In other words, our model is myopic, only caring about maximizing the points from the game. Perhaps coaches, and the league in general, are better long-term present value maximizers than our model as providing more entertainment maximizes "franchise value," even if slightly costly in terms of points. Particularly, if all teams do it the overall entertainment value of hockey is increased with no ex ante advantage or disadvantage to any one team as they're all acting alike. Now our analogy to business has drifted to collusion!

Finally, there is one bright spot. Coaches slowly come to adopt better strategies. It can take decades, but things do move in the rational direction. When this happens it proves that the stats geeks were right all along, but, more importantly, it also shows that it is possible to surmount the social and behavioral factors that too often sabotage optimal risk taking.

Suboptimal strategy in sports causes no net harm, and gives quants something to feel smug about, even if the jocks are still more popular. Suboptimal strategy in investing, however, can be a serious problem for everyone. Also, unfortunately, CIOs don't have the same advantages as coaches. They don't see the same situations repeated thousands of times, they have to try to make sense of a single investment history in rapidly changing financial markets. They don't get to start fresh at 0 – 0 every night, they need to think about five-year and ten-year and even longer histories, where a 5% loss this quarter means 5% less money forever.

One way to level the playing field is for CIOs to study risk carefully in laboratories like sports where the statistics are clearer but the human cognitive biases are essentially the same. That's why investors should care about when goalies are pulled.

And, coaches, you're really not pulling your goalies nearly early enough; well, at least not yet.

Appendix

In this appendix, we summarize key papers that seek to analyze the goalie pull. Only the last paper in Table 1 below (Zaman, 2001) produces predictions quantitatively similar to our own. The middle four papers all use the same data, with much higher scoring rates and much, much higher empty net scoring rates. The only one that our model directly compares to is the second paper (Erkut, 1987), and our model agrees reasonably closely if we use their data.

The first paper (Beaudoin et. al.) uses simulated hockey games. That allows the authors to make much more specific strategy decisions, including variables such as whether a team is at home or on the road, the penalty situation, where the puck is on the ice, and so forth. On the other hand, it requires far more assumptions about all aspects of the game.

The main reason that the authors of this first paper recommend waiting until later than our optimal time to pull in full-strength situations is because they consider the chance of getting a power play if you don't pull, and there can be more favorable earlier opportunities to score during power plays. Therefore, instead of a single optimal pull time for a team down by one goal, the authors recommend a complex strategy that could result in pulling earlier or later. Our view is that this recommendation may or may not give better hockey advice. It appears to be better, because their model includes many more factors and allows for a more complex recommendation. But our experience is that models with more parameters that recommend more complex strategies sometimes backfire. We'd like to see results from people implementing the strategies in real games, and unfortunately we'd need a lot of such evidence before fully trusting results of simulated games. In any case, for the purposes of our paper, it doesn't really matter because we're seeking general strategic insights; not specific hockey advice.

Table 1: Summary of Studies Focusing on the Goalie Pull

Author	Title	Citation	Data	Model	Optimal pull time with one goal deficit
Beaudoin, David and Swartz, Tim B.	Strategies for Pulling the Goalie in Hockey	White paper	NHL Website with assumptions	Simulation of games	3:00 for a road team when both teams are at full strength
Erkut, E.	Note: More on Morrison and Wheat's 'Pulling the Goalie Revisited'	Interfaces Vol. 17 No. 5, pp. 121-123, 1987.	NHL stats	Constant Poisson scoring probabilities for each team, with constant league-wide rate when goalie is pulled	1:59 - 3:81 depending on team scoring rate and whether playoff or regular season
Morrison, Donald G. and Wheat, Rita D.	Misapplications Reviews: Pulling the Goalie Revisited	Interfaces, Vol. 16, No. 6, Nov-Dec 1986, pp. 28-34	1978-79 NHL data on average goals scored per minute, plus MLE from number of empty net goals and number of minutes played without goalie	Constant Poisson scoring probabilities for each team, with and without pulled goalie	2:00 to 2:30

Nydick, Robert L. Jr. and Weiss, Howard J.	More on Erkut's "More on Morrison and Wheat's 'Pulling the Goalie Revisited'"	Interfaces Vol. 19 No. 5 Sept-Oct 1989, pp. 45-48	Erkut's data	Uses team-specific with-goalie scoring rates, and tests considers proportional rates with pulled-goalies, constant rate for all teams with pulled-goalies and intermediate models.	1:31 or earlier, possibly much earlier
Washburn, Alan	Still More on Pulling the Goalie	Interfaces Vol. 21 No. 2 March-Apr 1991, pp. 59-64	Erkut's data	Dynamic programming applied to Erkut's Model	1.71-4:00 depending on team scoring rate and whether playoff or regular season
Zaman, Zia	Coach Markov Pulls Goalie	Chance, Vol. 14, No. 2, 2001, pp. 31-35	NHL Yearbook, timing, simulation	Markov model with seven states based on puck position and ice action	8:07 with puck in offensive zone, 5:09 with puck in defensive zone

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Coffey, Wayne. *The Boys of Winter: The Untold Story of a Coach, a Dream, and the 1980 U.S. Olympic Hockey Team. 2005*. Crown Publishers, New York.

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